profiles of the free surface it is possible to ascertain the nature of streamlining for various bodies: under which conditions a given body allows representation by point poles, the determination of their orders and position, as well as under which conditions the "point" representation is violated. For example, we can study the case of the streamlining of a cylinder which is traditionally simulated by a point dipole. The streamlining potential for a solid is not an additive quantity [7], so that in the case in which the cylinder is in motion in the immediate vicinity of the free surface its influence is therefore significant, so that it is possible to assume that the potential of the dipole ceases satisfactorily to describe the streamlining of the cylinder. In view of the stability of the proposed method the vanishingly small errors in the measurement of the input data (the surface profile) introduce vanishingly small errors into the solution, i.e., the substitution of similar problems in the context of the proposed approach is valid.

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THE EVOLUTION OF WEAKLY LINEAR PERTURBATIONS IN A PLUG FORMED OF AN AIR—WATER MIXTURE

S. I. Lezhnin, I. I. Mullyadzhanov, V. E. Nakoryakov, B. G. Pokusaev, and N. A. Pribaturin

UDC 532.529

Various flow regimes arise in the motion of gas and vapor mixtures combined with liquids (a bubble flow, a plug-type flow, a rodlike flow, etc.), and these various types of flows are distinguished on the basis of their hydrodynamic and gasdynamic characteristics. At the present time, the formation and propagation of pressure waves in a mixture of a liquid with gas bubbles have been studied rather thoroughly, both from the theoretical and experimental standpoints. As regards the plug-type regime of flow in a gas—liquid mixture, existing information [1-4] is insufficient to comprehend the entire pattern involved in the process of wave formation. Initially, the model for the propagation of pressure waves was proposed independently in [3, 4], where it was assumed that the propagation of a wave in such a medium comes about as a result of inertialess compression and expansion of the gas plug and through the transfer of momentum to the liquid plug. It was demonstrated in [3, 5] that the mathematical description of the evolution of the waves is reduced, as in a bubble medium, to an equation of the Korteweg—de Vries type, and here we find also a hypothesis dealing with the possibility of forming pressure waves in such a medium, where the shape and quantitative relationships for the propagation of these waves are identical to those that prevail in a gas—liquid bubble mixture.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 30, No. 6, pp. 91-98, November-December, 1989. Original article submitted February 16, 1988; revision submitted August 5, 1988.

In the present paper we present a theoretical and experimental study of the formation and propagation of weakly linear ($\Delta p_0/p_0 < 1$) perturbations in pressure within a real (provision having been made for the aperiodicity of structure and phase slippage) gas–liquid mixture exhibiting a plug-flow structure.

Let us examine the unilateral propagation of a finite-duration low-amplitude pulse $\Delta p_0/p_0 < 1$ (p_0 is static pressure). Based on theoretical estimates [3], the velocity of propagation for a pulse through the plug flow of a two-phase mixture must be determined by means of the same formula as for a bubble mixture: $c_0 = [\gamma p_0/\rho_1\varphi(1-\varphi)]^{1/2}$ (γ is the adiabatic exponent of the gas, and φ is the volumetric gas content of the mixture). We will specify the initial width of the pulse $L = c_0t_0$ and its initial amplitude Δp_0. Here we will examine the wave for which $L > \ell$ (ℓ is the length of the two-phase cell of the plug formed by the liquid and a gas. If we introduce the dimensionless parameters $\tau = tc_0M/L$, $M = (\gamma + 1)\Delta p_0/2\gamma p_0$, $\eta = x/L$, $p^* = \Delta p/\Delta p_0$, then, as was demonstrated in [5], the evolution equation for waves in a gas–liquid plug-flow mixture is identical in form as for the case of a mixture of a liquid with gas bubbles:

$$\frac{\partial p^*}{\partial \tau} + \frac{\partial p^*}{\partial \eta} + p^*\frac{\partial p^*}{\partial \eta} + \frac{1}{\sigma^2}\frac{\partial^3 p^*}{\partial \eta^3} = 0 \tag{1}$$

[$\sigma^2 = M(24L^2/\ell^2)$, σ, M are dimensionless criteria of similarity].

It was noted in [3] that a unique feature of plug-type flow is the appearance of three-dimensional dispersion in the propagation through that flow of perturbation in pressure. The limit signal-propagation frequency $2\omega_0 = 2c_0/\ell = 2[\gamma p_0/\rho_1\ell^2\varphi(1-\varphi)]^{1/2}$. It follows from a comparison of the resonance frequencies ω_0 for a bubble medium [2] and for a plug-type mixture that the characteristic pulsation frequencies in wave propagation through the mixture under consideration must be low in value (on the order of several tens of hertz, and in the case of a bubble stream, on the order of kilohertz). Knowing these criteria, it is easy to determine the paths along which the initial perturbation develops. Estimates show that for a plug regime in tubes with a diameter from 8 to 30 mm the value of σ is small and the contribution of the dispersion effects to the formation of the pressure wave is considerably greater than in the case of a bubble flow regime.

From the standpoint of an experimental verification of the theoretical model for wave propagation, the shock-tube method proves to be most convenient. In our studies these experiments were conducted in a tube with a vertical working section 25 mm and 8 mm in diameter with a height of 2.5 m and 0.8 m, respectively. Here the two-phase medium plug structure was formed through the periodic supply of a specific amount of gas into the working section in conjunction with a nonmoving liquid for which distilled water was used. Such a method of introducing the gas enabled us to obtain a pure plug structure (without gas bubbled in the liquid slug), exhibiting regulated characteristics ($\varphi = 0.1$-0.7, $\ell_{20}/D \sim 1$-10, ℓ_{20} is the initial length of the gas plug and D represents the diameter of these plugs). We should note that it is all the more difficult to increase the velocity at which these plugs move (to increase φ), the greater the diameter of the channel. Under certain conditions associated, as was demonstrated in [1], with agitation of the liquid, each plug enters the wake of the previous plug and depending on buoyancy overtakes the earlier plug and merges together with it. For the experiments with a channel 25 mm in diameter, the maximum attainable values of the gas content were 0.3-0.4. The pressure waves are generated by the opening of an electromagnetic valve separating the high-pressure chamber from the low-pressure chamber. The required duration of perturbation (15-500 msec in our experiments) was achieved by opening the valve for a specific time interval. The amplitude of the initial wave varied in the range $(0.015$-$3.1)p_0$ by changing the pressure within the high-pressure chamber of the shock tube. Thus, the criteria σ and M were varied in the experiments. The propagation of the pressure perturbations through the two-phase medium were recorded by piezoelectric pressure sensors. The content of gas and the dimensions of the plug were measured by a time-passage method for which two photodiodes were used, the latter mounted at a distance of 30 mm from each other. The experimental installation was controlled and the collection and processing of data were all accomplished with an "Elektronika-60" computer with an expanded KAMAK complex (ADC, a switching unit, a timer, and a counter).

Figure 1 shows the velocities of the weak compression waves ($\Delta p_0/p_0 < 1$) as a function of φ. Points 1 and 2 correspond to experiments with D = 8 and 25 mm, while line 3 represents the calculated relationship between c_0 and φ for an ideal structure. At D = 25 mm the maximum $\varphi = 0.14$, and here a stable flow regime exists. No provision was made in our

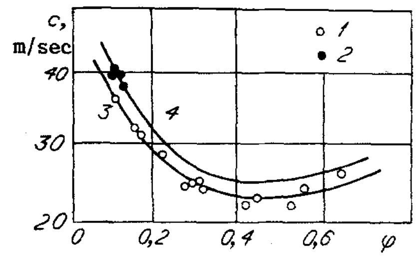

Fig. 1

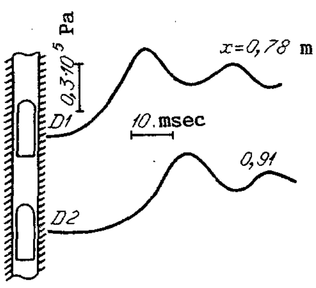

Fig. 2

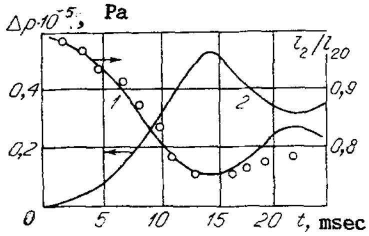

Fig. 3

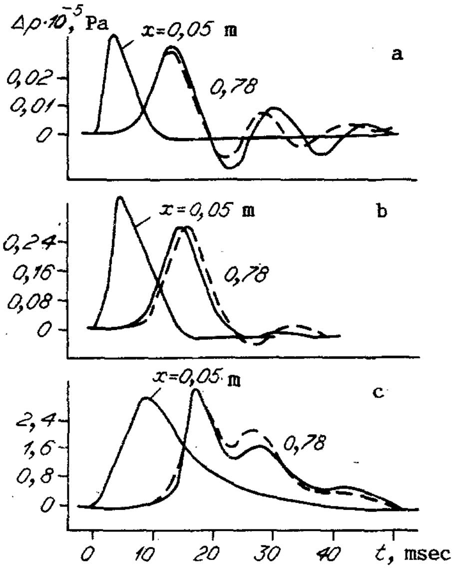

Fig. 4

installation for the possibility of varying the scattering in the plug lengths. At the same time, with steady-state plug flow in a tube with D = 8 mm we had a considerably smaller dispersion in the distribution of plug lengths ($\delta\ell_{20}$ = 0.5 mm with ℓ_{20} = 50 mm) in comparison with a tube for which D = 25 mm ($\delta\ell_{20}$ = 15 mm when ℓ_{20} = 50 mm). The deviation of the velocity of propagation for the weak compression waves in the case of D = 25 mm from the value of c_0 is in rather good agreement with the calculations presented below [relationship (21)], in which provision is made for the influence exerted by dispersion on the distribution of plug lengths.

Measurements of the propagation velocity for small perturbations showed that the results were independent of the plug length for the same φ. When the two-phase medium is acted on by a high-frequency perturbation exhibiting the characteristic frequencies that are larger than the resonance frequency of the plug, we observed no advance indicators moving through the mixture at a velocity greater than c_0. In this case, the first gas plug appears, however, as a shield against the high-frequency perturbations.

Results from a synchronous recording of the oscillations in two adjacent plugs provide a clear representation of the formation of a pressure wave in such a medium (Fig. 2, $\Delta p_0/p_0$ = 0.5, φ = 0.3, ℓ_{20} = 40 mm, D = 8 mm) and the pressure wave that arises in the vicinity of the gas plug is shown in Fig. 3. We can see from Fig. 2 that the oscillations of the two adjacent plugs occur with a phase shift relative to one another. The behavior of the air plug in the pressure wave is nearly adiabatic, i.e., $p_2 = p_0(\ell_{20}/\ell_2)^\gamma$ (Fig. 3, line 1 represents the calculation, while 2 represents the profile of the pressure in the wave; the circles identify the behavior of the plug in the experiment, ℓ_{20} = 60 mm, D = 25 mm). This is quite natural here, since the characteristic time required to equalize the temperature within a plug of diameter D and with a thermal gas diffusivity is expressed as $a_2\tau = D^2/4\pi a_2 \approx 2$ sec. However, in the general case of analyzing the wave dynamics we must examine the possible heat losses. For a plug-flow regime this involves, first of all, the exchange of heat between the gas and the liquid and between the liquid and the wall. If the thickness of the

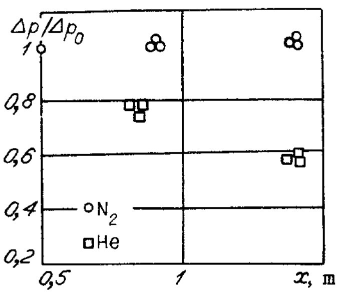

Fig. 5

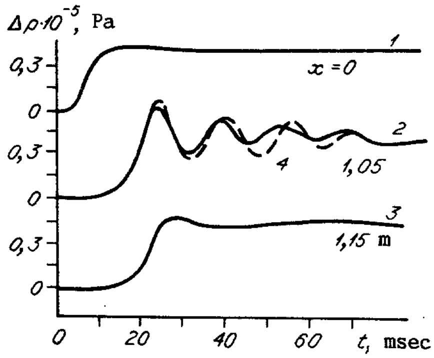

Fig. 6

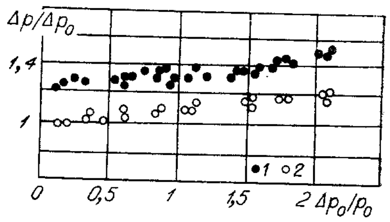

Fig. 7

liquid film between the plug and the wall of the channel is extremely small, and if the wall of the channel exhibits slow thermal conductivity, the condition of temperature constancy for the gas may be disrupted. In the solution of the internal thermal problem in a gas plug, unlike a bubble, substantially convective flows may exist.

Even without taking these into consideration, the characteristic time required to equalize the temperature in the plug is small and comparable to the duration of the wave front. Thus, in the case of a helium plug in a tube with D = 8 mm, $\tau = 0.03$ sec.

Let us examine the propagation of a pressure pulse in the plug-flow regime of an air—water stream. As we can see from Fig. 4 ($\varphi = 0.18$, D = 25 mm, $\ell_{20} = 65$ mm), the structure of the pressure waves which arise in the medium indeed depend both on σ and M. Just as in a bubble regime for a two-phase mixture, there exists such a value of $\sigma = \sigma_*$, governed by the form of the initial perturbation (for example, for perturbations Gaussian in form, i.e., $\sigma_* = 3.1$, or for a triangular wave with $\sigma_* = 13.1$), beneath which a steady-state wave packet is formed (Fig. 4a, $\sigma = 2.9$). With $\sigma = 7.4$ (in our experiments $\sigma_* \simeq 6$) a soliton forms in the medium (Fig. 4b) whose shape is described by the expression $\Delta p = \Delta p_m \operatorname{sech}^2(x/\delta)$, $\delta = \ell[(\Delta p_0/p_0)(\gamma + 1)/\gamma]^{-1/2}$. The amplitude of the soliton is kept virtually constant (in the case of nitrogen plugs) over the entire length of the working section (Fig. 5, D = 25 mm, $\varphi = 0.16$, $\ell_{20} = 58$-68 mm, $\sigma = 6.5$). On the other hand, if we are dealing with a gas capable of greater heat conduction, such as, for example, helium, then on the basis of that which was stated earlier, in the propagation of the wave we have a reduction in its amplitude, which can be seen clearly in Fig. 5. Figure 4c ($\sigma = 27$) illustrates the fact that when $\sigma > \sigma_*$ the initial perturbation evolves in the form of a nonlinear wave with an oscillating trailing front.

Let us take a look at the propagation of a shock wave (SW) in a medium. As we can see from Figs. 2 and 6 (line 2, $p_0 = 10^5$ Pa, $\varphi = 0.18$, $\ell_{20} = 60$ mm, D = 25 mm), in the region of the gas plugs the measured pressure profile exhibits an oscillating structure. For weak waves $\Delta p_0/p_0 \lesssim 0.2$ the oscillation frequency corresponds approximately to the resonance frequency ω_0, while the width of the leading front corresponds to half of ω_0. With an increase in the intensity of the wave the width of the leading front is diminished, primarily as a consequence of the appearance of nonlinear effects. The amplitude of the SW increases with an increase in the initial intensity of the wave and with $\Delta p_0/p_0 > 0.5$ may exceed the initial intensity by a factor of 1.4 (Fig. 7, points 1, $p_0 = 10^5$ Pa, $\varphi = 0.2$, $\ell_{20} = 60$ mm, D = 25 mm). The velocity of the SW corresponds well to the familiar expression $u/c_0 = 1 + (\gamma + 1)\Delta p_0/p_0$. In an air—water mixture the SW, fixed within the region of gas plugs, retains its parameters over the entire length of the working section (for D = 25 mm and H = 2.5 m, H is the length of the working section). In the plug-flow structure of the two-phase mixture, since we are dealing with a discrete spatial distribution of liquid and gas, it is quite natural to expect differences in wave shape within a single two-phase cell such as that formed by the gas plug and the liquid slug. Since two adjacent plugs oscillate with a shift in phase (see Fig. 2), and the pressure within the liquid plug separating these plugs changes linearly along the slug, from the pressure value within a single plug to the pressure values in the other, the oscillation in pressure within the liquid slug will be less clearly defined than in the gas plug. As we can see from the experimental results shown in Fig. 6 (curve 3), the SW fixed in the liquid slug oscillates weakly, and its maximum magnitude virtually coincides with the pressure in the incident wave (Fig. 7, points 2).

It follows from the experimental results presented that the fundamental positions of the mathematical model, based on an idealized scheme [3], have been confirmed experimentally. However, in a real flow the gas plugs do not occupy the entire cross section of the tube and, moreover, in a chain of such plugs their dimensions are not kept constant. Before we compare the experimental and theoretical results, we have to clarify the question as to the influence exerted by the "nonideal" structure on the wave characteristics. If we treat the gas as an ideal gas with an adiabatic exponent γ, we can neglect the friction of the liquid against the channel wall, as well as the interphase heat exchange, and thus we can write the equation of motion for the n-th liquid slug and the equation of state for the n-th gas plug, as follows:

$$\rho_1 l_{1,n}\frac{d^2x_n}{dt^2} = p_n - p_{n+1}; \tag{2}$$

$$p_n = p(0)\left(1 + \frac{x_n - x_{n-1} - l_{2,n-1}}{l_{2,n}}\right)^{-\gamma}. \tag{3}$$

In a real flow, since the length of the plugs and the liquid slugs fluctuate about a mean value, we can represent $\ell_{1,n}$ and $\ell_{2,n}$ in the following form:

$$l_{1,n} = (1-\varphi)l(1+\alpha_n); \tag{4}$$

$$l_{2,n} = \varphi l(1+\beta_n) \tag{5}$$

(α_n, β_n are the dimensionless small random quantities with an average zero value and φ is the value of the volumetric gas content averaged over the entire length of the plug structure).

We have adopted the following model: α_n, β_n are random quantities independent with respect to the natural argument n, with a zero correlation radius:

$$\overline{\alpha_n\alpha_k} = \alpha^2\delta_{nk},\ \overline{\beta_n\beta_k} = \beta^2\delta_{nk},\ \overline{\alpha_n\beta_k} = 0. \tag{6}$$

In order to determine the change in the fundamental characteristics of the signal as it is propagated through a real mixture, we will derive the corresponding wave equation. From system (2)-(5) we obtain a chain of difference equations

$$\frac{d^2}{d\tau^2}\left[\delta p_n - \frac{\gamma+1}{2\gamma p_0}(\delta p_n)^2\right] = (1+\beta_n)^{-1}\left[\frac{\delta p_{n+1}}{\alpha_n+1} + \frac{\delta p_{n-1}}{\alpha_{n-1}+1} - \delta p_n\frac{2+\alpha_n+\alpha_{n-1}}{(1+\alpha_n)(1-\alpha_{n-1})}\right]. \tag{7}$$

It follows from this that if in the fluctuations of the dimensions of the plug and the liquid slug the overall length of the cell is retained, then the frequency of the signal will not change. The pressure perturbation δp_n is represented as the sum of the long-wave averaged component $\bar{p}_n$ and the random component p_n', whose value may change significantly when n changes by unity. Here $p_n' \ll \bar{p}_n$, since α_n, $\beta_n \ll 1$. In the following we will examine only the evolution of the low-frequency perturbations ($\omega \ll \omega_0$). Expanding Eq. (7) over the small parameters α_n, β_n and averaging it over n, with consideration of (5) accurate to second-order terms of smallness, we will obtain

$$\frac{d^2}{d\tau^2}\left[\bar{p}_n - \frac{\gamma+1}{2\gamma p_0}(\bar{p}_n)^2\right] = (\bar{p}_{n+1} - 2\bar{p}_n + \bar{p}_{n-1})(1+\alpha^2+\beta^2) - \overline{p_n'(\alpha_n+\beta_n)} + \overline{p_n'(2\beta_n+\alpha_n+\alpha_{n-1})} - \overline{p_{n-1}'(\alpha_{n-1}+\beta_n)} \tag{8}$$

for the averaged quantity $\bar{p}_n$,

$$\frac{d^2}{d\tau^2}p_n' = p_{n+1}' - 2p_n' + p_{n-1}' - \beta_n(\bar{p}_{n+1} - 2\bar{p}_n + \bar{p}_{n-1}) + \alpha_n(\bar{p}_n - \bar{p}_{n+1}) - \alpha_{n-1}(\bar{p}_n - \bar{p}_{n-1}) \tag{9}$$

for the random component p_n'. In the solution of the linear nonuniform equation (9) we find the unknown quantities $\bar{p}_n$. Substituting (9) into (8) and carrying out the averaging operation, we are left with a single equation for the function $\bar{p}_n$. By means of the Fourier transform over time, we can present relationship (9) as follows:

$$p_{n+1}' - (2-\omega^2)p_n' + p_{n-1}' = A_n. \tag{10}$$

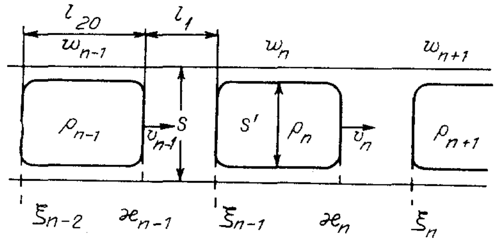

Fig. 8

It is easy to find the particular solution of Eq. (10):

$$p'_n = \sum_{-\infty}^{n} \chi \frac{\sin (n-m)\Psi}{\sin \Psi} A_m - \sum_{n}^{+\infty} (1-\chi) \frac{\sin (n-m)\Psi}{\sin \Psi} A_m$$

$\omega^2 = 4\sin^2(\Psi/2)$, χ is an arbitrary constant.

From formulas (6) and (10) we obtain

$$\begin{gathered}\overline{p'_n\beta_n} = 0,\ \overline{p'_n\alpha_n} = \chi\alpha^2(\overline{p_{n+1}} - \overline{p_n}),\ \overline{p'_n\alpha_{n-1}} = (1-\chi)\,\alpha^2(\overline{p_{n-1}} - \overline{p_n}), \\ \overline{p'_{n-1}\alpha_{n-1}} = \chi\alpha^2(\overline{p}_n - \overline{p}_{n-1}),\quad \overline{p'_{n-1}\beta_n} = \chi\beta^2(-\overline{p}_{n+1} + 2\overline{p}_n - \overline{p}_{n-1}), \\ \overline{p'_{n+1}\alpha_n} = (1-\chi)\,\alpha^2(\overline{p}_n - \overline{p}_{n+1}),\quad \overline{p'_{n+1}\beta_n} = \\ = (1-\chi)\,\beta^2(-\overline{p}_{n+1} + 2\overline{p}_n - \overline{p}_{n-1}).\end{gathered} \tag{11}$$

Since ω, $\Psi \ll 1$ (low-frequency perturbations), in the averaging we took the kernel $\sin[(n - m)\Psi]$ and p_n to be constants. Substituting (11) into (8), we find

$$\left(\overline{p}_n - \frac{\gamma+1}{2\gamma p_0}\overline{p}_n^2\right)_{\tau\tau} = (\overline{p}_{n+1} - 2\overline{p}_n + \overline{p}_{n-1})(1 + 2\alpha^2 + 2\beta^2). \tag{12}$$

Just as in [3], for the transition from the discrete argument to the function of the continuous argument, we will use the mathematical model of a quasicontinuum [6]. The differential equation corresponding to system (12), for the function of the continuous argument in the long-wave approximation, will then be written in the form of the Boussinesq equation

$$\left(\delta p - \frac{\gamma+1}{2\gamma p_0}(\delta p)^2\right)_{\tau\tau} - c_0^2(1 + 2\alpha^2 + 2\beta^2)\left(\delta p_{xx} - \frac{l^2}{12}\delta p_{xxxx}\right) = 0.$$

It follows from this that with an "aperiodic" plug structure for the flow the waves will propagate at a velocity greater by a factor of $\sqrt{1 + 2\alpha^2 + 2\beta^2}$ than in the case of a "periodic" structure exhibiting the same volumetric gas content.

The following correction factor to the model, under consideration here, of the propagation of weak waves is associated with consideration of the thin liquid film surrounding the gas plug. For a vertical flow the existence of such a film leads to the development of a difference in the average velocities of liquid and gas motion, which must primarily affect the change in the velocities of propagation for the perturbations. Since consideration of phase slippage leads to complex notation of the equation of motion (1), the fundamental features in the changes for the quantitative relationships governing the propagation of a wave can be analyzed through the Lagrange function.

We will assume that because of the limited density of the gas (insignificant pressure gradients along the plug) during the propagation of the wave the average velocity and film thickness θ, averaged over the cross section of the film, do not change [$\theta = (S - S')/S$ = const (Fig. 8)]. We will denote the displacement of the right-hand boundary of the liquid slug by ξ_n and with κ_n we denote the displacement of the left-hand boundary of the liquid slug, and v_n denotes the velocity of the flow in the n-th slug, with w_n denoting the velocity of the liquid in the n-th film along the n-th plug. Using a mechanical analogy [3], we will write the Lagrange function of the system

$$\mathscr{L} = \sum_n \frac{m_n v_n^2}{2} + \frac{m_{fn} w_n^2}{2} \frac{p_0 S l_2}{\gamma - 1}(1-\theta)\left(1 + \frac{\xi_n - \varkappa_{n-1}}{l_2}\right)^{1-\gamma} \tag{13}$$

($m_n = \rho_1 S \ell_1 [1 + (\xi_n - \kappa_n)/\ell_1]$ is the mass of the film in the n-th plug). Using a cylinder to simulate the gas plug, from the condition of continuity for the liquid phase we find the relationship between ξ, $\dot{\kappa}$, v, w.

Turning to a system associated with the motion of the interphase boundary, we obtain

$$v_n - \dot{\xi}_n = \theta(w_n - \dot{\xi}_n); \tag{14}$$

$$v_n - \dot{\varkappa}_n = \theta(w_{n+1} - \dot{\varkappa}_n); \tag{15}$$

$$v_{n-1} - \dot{\varkappa}_{n-1} = \theta(w_n - \dot{\varkappa}_{n-1}). \tag{16}$$

Subtracting (16) from (14) and subtracting (14) from (16) we obtain:

$$v_n = v_{n-1} + (1-\theta)(\dot{\xi}_n - \dot{\varkappa}_{n-1}); \tag{17}$$

$$w_{n+1} = w_n + \frac{(1-\theta)}{\theta}(\dot{\xi}_n - \dot{\varkappa}_n). \tag{18}$$

We have earlier made the assumption that the velocity of the liquid in the film does not change in the wave process ($w_n = w_0 = \text{const}$). Consequently, from (18) we have $\dot{\xi}_n = \dot{\kappa}_n$, $\xi_n = \kappa_n$ (the mass of the slug does not change). Using formula (17) as a recursion formula, we write $v_n = v_{-\infty} + (1 - \theta)(\dot{\xi}_n - \dot{\xi}_{-\infty})$. The Lagrange equation (13) is presented in the form

$$\mathscr{L} = \sum_n \frac{\rho_1 S l_1}{2}(v_{-\infty} + (1-\theta)(\dot{\xi}_n - \dot{\xi}_{-\infty})]^2 + \\ + \frac{\theta \rho_1 S l_2}{2}\left(1 + \frac{\xi_n - \varkappa_{n-1}}{l_2}\right)\frac{w_0^2}{2} - \frac{p_0 S l_2}{\gamma-1}(1-\theta)\left(1 + \frac{\xi_n - \varkappa_{n-1}}{l_2}\right)^{1-\gamma}. \tag{19}$$

The equation for the dynamics of the mechanical system exhibiting the Lagrange function $\mathscr{L}(\xi_n, \dot{\xi}_n)$ is well known:

$$(d/dt)(\partial\mathscr{L}/\partial\dot{\xi}_n) = \partial\mathscr{L}/\partial\xi_n. \tag{20}$$

Assuming that $v_{-\infty} = \dot{\xi}_{-\infty} = 0$ and examining the small perturbations, from (19) and (20) we find

$$\ddot{\xi}_n = \frac{\gamma p_0}{\rho_1 l_1 l_2 (1-\theta)}(\xi_{n+1} - 2\xi_n + \xi_{n-1}).$$

By turning to the continual analog of this equation we determine the velocity of propagation for the perturbations

$$c_0' = \sqrt{\frac{\gamma p_0 (l_1 + l_2)}{\rho_1 l_1 l_2 (1-\theta)}} = c_0 (1-\theta)^{-1/2}.$$

With consideration of the aperiodicity of the structure

$$c_* \simeq c_0 (1 + \alpha^2 + \beta^2 + \theta/2). \tag{21}$$

The value of c_0' is greater by a factor of approximately 3% than the value of c_0 calculated without consideration of the presence of a liquid film. This addition is small in comparison with the change introduced due to the scattering in the plug lengths, and in experiments with water it has not been established. We note that in the case of a water–glycerine solution ($\nu \simeq 0.9 \cdot 10^{-3}$ m²/sec when T = 293 K) $\theta \simeq 0.2$ and the contribution from the effect of the liquid film on c_0' reaches ≈11%. The increase in the wave velocity due to various plug lengths amounts to 18% (points 2 in Fig. 1), which is within the scope of the values of c_* (Fig. 1, line 4), determined from formula (21).

Thus, in the calculations of wave evolution we can use Eq. (1) with the corresponding correction factor for c_0. Equation (1) was calculated on the basis of the difference scheme proposed in [7]. The profile of the initial pressure perturbation served as the initial conditions. The theoretical values for the pressure waves were determined at distances corresponding to the locations of the pressure sensors in the experimental installation. As we can see from Fig. 4 (the dashed line represents the theoretical values), the agreement between the experimental and theoretical results is satisfactory.

The structure of the SW in the region of the gas plug can be calculated on the basis of the Burgers–Korteweg–de Vries model. Unfortunately, we currently have no model which allows us adequately to calculate the coefficient of effective viscosity in the plug-flow regime of a two-phase mixture. If we formally extend the results from the analysis of the SW in a bubble flow [2] to plug flow (apparently this is possible since the pressure pulses in this medium evolve in the same manner as in a mixture of a liquid with gas bubbles), $(1/\mathrm{Re})(\partial^2 p^*/\partial\eta^2)$ must be added to the left-hand side of Eq. (1) (Re contains only the kinematic viscosity ν). Proceeding in the same manner as in the case of a mixture of liquid and gas bubbles, we can find some critical value of $\nu_* = [(\ell^2 c_0{}^2 \Delta p/12p_0)(2\gamma/(\gamma + 1))]^{1/2}$. On fulfillment of the condition $\nu < \nu_*$ in the medium an oscillating SW must appear. From our evaluations it is evident that the condition is always valid for a plug-flow regime. Calculation of the SW on the basis of the BK-dV equation with $\nu = 10^{-6}$ m^2/sec is illustrated in Fig. 6 (line 4).

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